Further Differentiation – Trig and Chain Rule

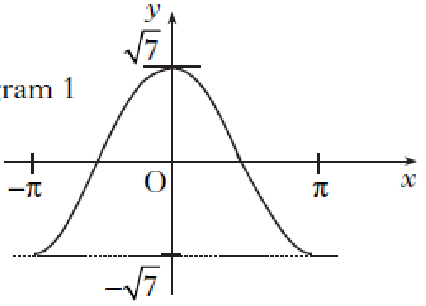
2000 P2	**B8.** Given that $f(x) = (5x-4)^{\frac{1}{2}}$, evaluate $f'(4)$.	**3**
2002 P2	**6.** Find the equation of the tangent to the curve $y = 2\sin\left(x - \frac{\pi}{6}\right)$ at the point where $x = \frac{\pi}{3}$.	**4**
5. (JAN) 02 P1	A function f is defined by $f(x) = 2x + 3 + \frac{18}{x-4}$, $x \neq 4$. Find the values of x for which the function is strictly increasing.	**5**
2003 P2	**6.** If $f(x) = \cos(2x) - 3\sin(4x)$, find the exact value of $f'\left(\frac{\pi}{6}\right)$.	**4**
2004 P1	**6.** Given that $y = 3\sin(x) + \cos(2x)$, find $\frac{dy}{dx}$.	**3**
2005 P1	**5.** Differentiate $(1 + 2\sin x)^4$ with respect to x.	**2**
2006 P1	**5.** A function f is defined by $f(x) = (2x-1)^5$. Find the coordinates of the stationary point on the graph with equation $y = f(x)$ and determine its nature.	**7**
2006 P2	**9.** If $y = \frac{1}{x^3} - \cos 2x$, $x \neq 0$, find $\frac{dy}{dx}$.	**4**
2007 P1	**10.** Given that $y = \sqrt{3x^2+2}$, find $\frac{dy}{dx}$.	**3**

2008
P2

3. (*a*) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$.

Write down the value of p.

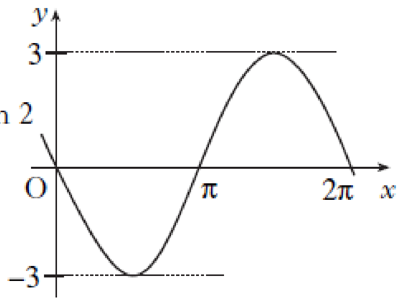
Diagram 1



(ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$.

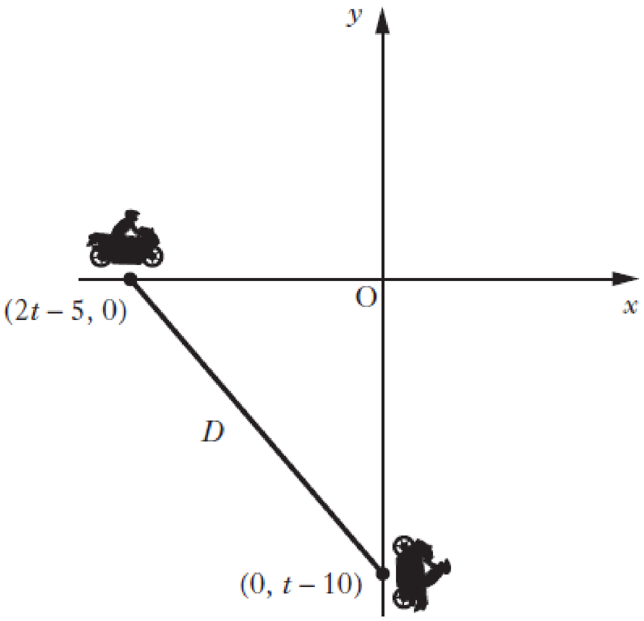
Write down the value of q. **2**

Diagram 2

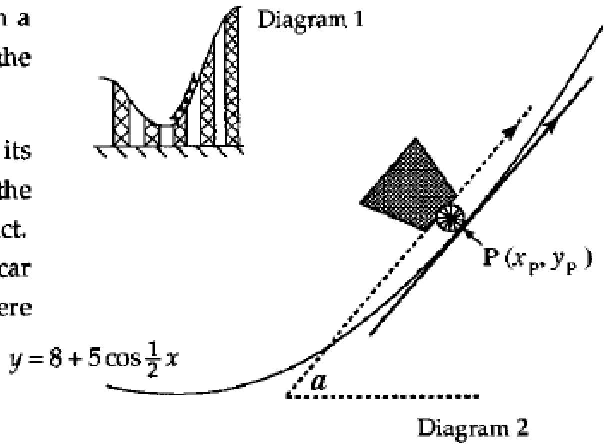
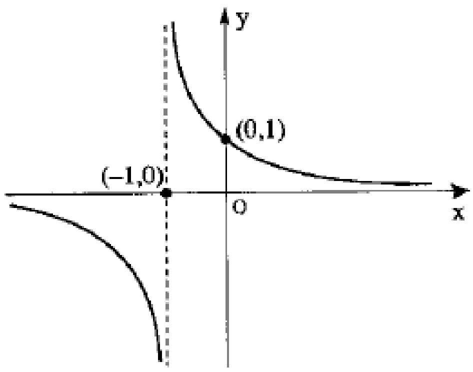


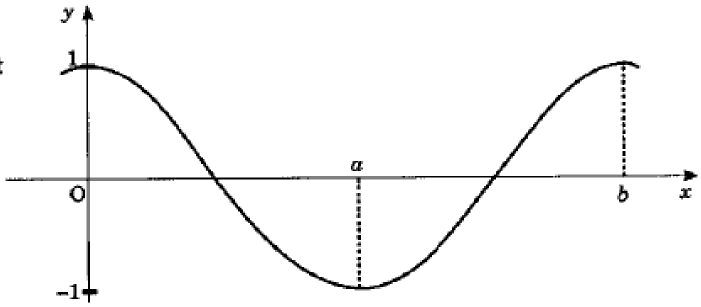
(*b*) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. **4**

(*c*) Hence find $f'(x) + g'(x)$ as a single trigonometric expression. **2**

2015 OLD P1	**25.** A stunt performed by two members of a motorcycle display team requires them to travel, at speed, at right angles to each other across the arena.  The positions of the motorcyclists, relative to suitable axes, t seconds after the stunt begins, are $(2t - 5, 0)$ and $(0, t - 10)$. (*a*) Show that, at any given moment, the distance, D, between them is given by $D = \sqrt{5t^2 - 40t + 125}$. **2** (*b*) Determine whether the distance between the motorcyclists is increasing or decreasing 5 seconds after the start of the stunt. **4**
2015 SP P1	**10.** Find the rate of change of the function $f(x) = 4\sin^3 x$ when $x = \frac{5\pi}{6}$. **3**
2016 P2	**11.** (a) Show that $\sin 2x \tan x = 1 - \cos 2x$, where $\frac{\pi}{2} < x < \frac{3\pi}{2}$. **4** (b) Given that $f(x) = \sin 2x \tan x$, find $f'(x)$. **2**
2017 P1	**3.** Given $y = (4x - 1)^{12}$, find $\frac{dy}{dx}$. **2**
2017 P2	**11.** (a) Show that $\frac{\sin 2x}{2\cos x} - \sin x \cos^2 x = \sin^3 x$, where $0 < x < \frac{\pi}{2}$. **3** (b) Hence, differentiate $\frac{\sin 2x}{2\cos x} - \sin x \cos^2 x$, where $0 < x < \frac{\pi}{2}$. **3**

Pre 2000 - Further Differentiation

1	Given that $f(x) = 5(7-2x)^3$, find the value of $f'(4)$.	**4**
2	Find the derivative, with respect to x, of $\frac{1}{x^3} + \cos 3x$.	**4**
3	Differentiate $4\sqrt{x} + 3\cos 2x$.	**4**
4	Differentiate $\sin 2x + \frac{2}{\sqrt{x}}$ with respect to x.	**4**
5	Find $\frac{dy}{dx}$ given that $y = \sqrt{1+\cos x}$.	**3**
6	Differentiate $2x^{\frac{3}{2}} + \sin^2 x$ with respect to x.	**4**
7	If $f(x) = \cos^2 x - \frac{2}{3x^2}$, find $f'(x)$.	**4**
8	Given $f(x) = \cos^2 x - \sin^2 x$, find $f'(x)$.	**3**
9	Given $f(x) = (\sin x + 1)^2$, find the exact value of $f'\left(\frac{\pi}{6}\right)$.	**3**
10	Diagram 1 shows 5 cars travelling up an incline on a roller-coaster. Part of the roller-coaster rail follows the curve with equation $y = 8 + 5\cos\frac{1}{2}x$. Diagram 2 shows an enlargement of the last car and its position relative to a suitable set of axes. The floor of the car lies parallel to the tangent at P, the point of contact. Calculate the acute angle a between the floor of the car and the horizontal when the car is at the point where $x_P = \frac{7\pi}{3}$. Express your answer in degrees.  	**4**
11	The diagram shows the graph of the function $$f(x) = \frac{1}{x+1}, \quad x \neq -1.$$ **Prove** that the function f is decreasing for all values of x except $x = -1$.  	**4**

12	Part of the graph of $y = f(x)$ is shown in the diagram. This graph has stationary points at $x = 0$, $x = a$ and $x = b$. (a) Sketch the graph of $y = f'(x)$ for $0 \le x \le b$. (b) If $a = \pi$ and $b = 2\pi$, write down a possible expression for $f'(x)$. 	**3, 1**

Further Integration – Trig and Reverse Chain Rule

Indefinite Integral

2000 P2	**B10.** Find $\int \frac{1}{(7-3x)^2}\,dx$.	*Marks* **2**
2015 OLD P2	7. (a) Find $\int (3\cos 2x + 1)\,dx$.	2
	(b) Show that $3\cos 2x + 1 = 4\cos^2 x - 2\sin^2 x$.	2
	(c) Hence, or otherwise, find $\int (\sin^2 x - 2\cos^2 x)\,dx$.	2
2015 P2	7. (a) Find $\int (3\cos 2x + 1)\,dx$.	2
	(b) Show that $3\cos 2x + 1 = 4\cos^2 x - 2\sin^2 x$.	2
	(c) Hence, or otherwise, find $\int (\sin^2 x - 2\cos^2 x)\,dx$.	2
2016 P1	5. Find $\int 8\cos(4x+1)\,dx$.	2
2017 P1	13. Find $\int \frac{1}{(5-4x)^{\frac{1}{2}}}\,dx,\ x < \frac{5}{4}$.	4

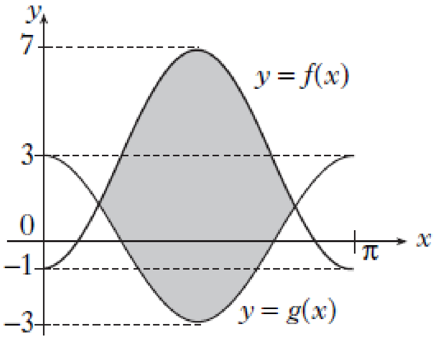
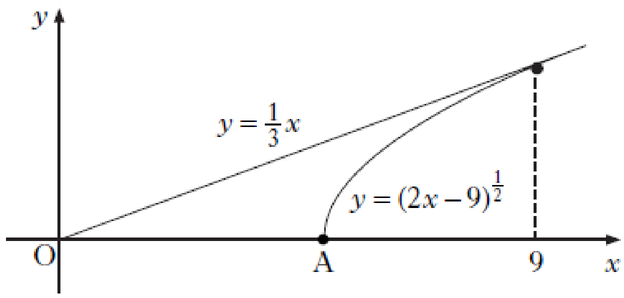
Evaluating a Definite Integral

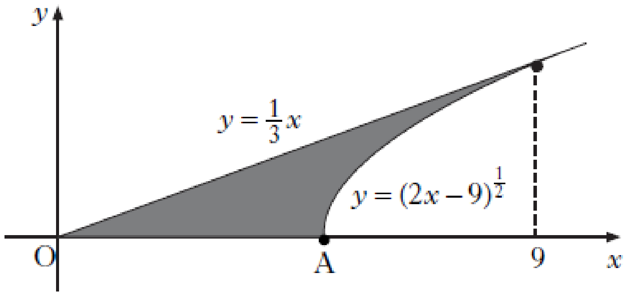
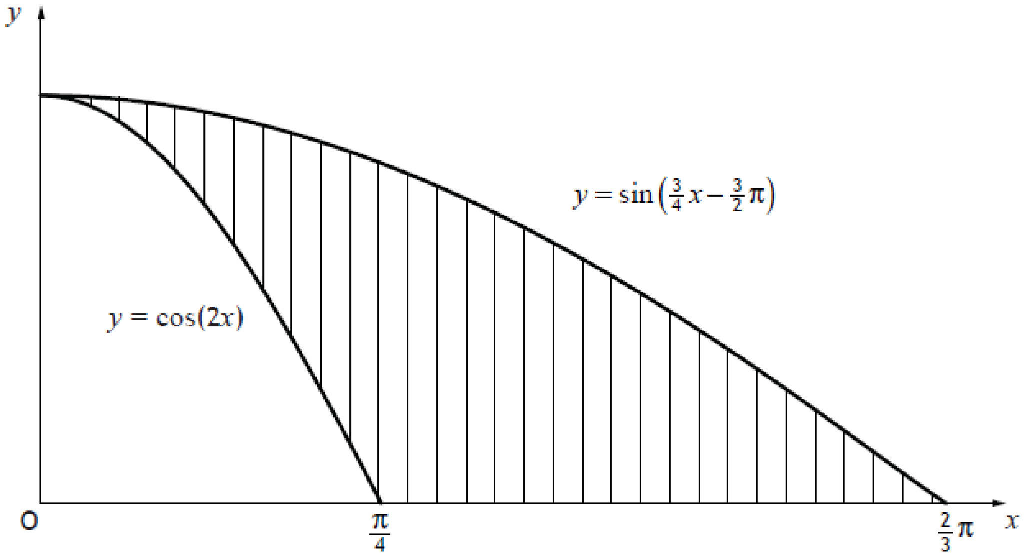
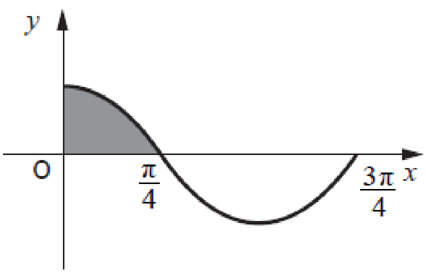
6. (JAN) 02 P2	Find $\int_0^1 \left(\cos(3x) - \sin\left(\frac{1}{3}x + 1\right)\right)dx$ correct to 3 decimal places.	**3**
2003 P1	**8.** Find $\int_0^1 \frac{dx}{(3x+1)^{\frac{1}{2}}}$.	**4**
2004 P1	**7.** Find $\int_0^2 \sqrt{4x+1}\,dx$.	**5**
2007 P2	7. Find the value of $\int_0^2 \sin(4x+1)\,dx$.	**4**

Finding Limits of a Definite Integral

2011 P2	**6.** (*a*) The expression $3\sin x - 5\cos x$ can be written in the form $R\sin(x+a)$ where $R>0$ and $0 \le a < 2\pi$. Calculate the values of R and a.	**4**
	(*b*) Hence find the value of t, where $0 \le t \le 2$, for which $$\int_0^t (3\cos x + 5\sin x)\,dx = 3.$$	**7**
2013 P2	**6.** Given that $\int_0^a 5\sin 3x\,dx = \frac{10}{3}$, $0 \le a < \pi$, calculate the value of a.	5
2014 P2	**5.** Given that $\int_4^t (3x+4)^{-\frac{1}{2}}\,dx = 2$, find the value of t.	**5**
2015 EP P2	**8.** Given that $\int_{\frac{\pi}{8}}^{a} 5\sin(4x - \frac{\pi}{2})dx = \frac{10}{4}$, $0 \le a < \frac{\pi}{2}$, calculate the value of a.	**6**

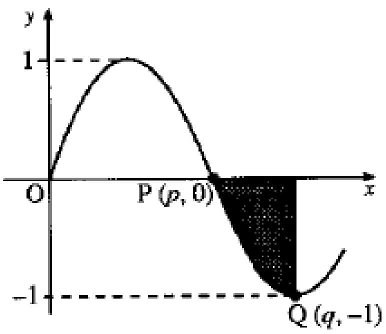
Calculating Areas

2009 P2	**5.** The graphs of $y = f(x)$ and $y = g(x)$ are shown in the diagram. $f(x) = -4\cos(2x) + 3$ and $g(x)$ is of the form $g(x) = m\cos(nx)$. 	
	(*a*) Write down the values of m and n.	**1**
	(*b*) Find, correct to one decimal place, the coordinates of the points of intersection of the two graphs in the interval $0 \le x \le \pi$.	**5**
	(*c*) Calculate the shaded area.	**6**
2010 P2	**6.** (*a*) A curve has equation $y = (2x-9)^{\frac{1}{2}}$. Show that the equation of the tangent to this curve at the point where $x = 9$ is $y = \frac{1}{3}x$.	**5**
	(*b*) Diagram 1 shows part of the curve and the tangent. The curve cuts the x-axis at the point A.  *Diagram 1* Find the coordinates of point A.	**1**

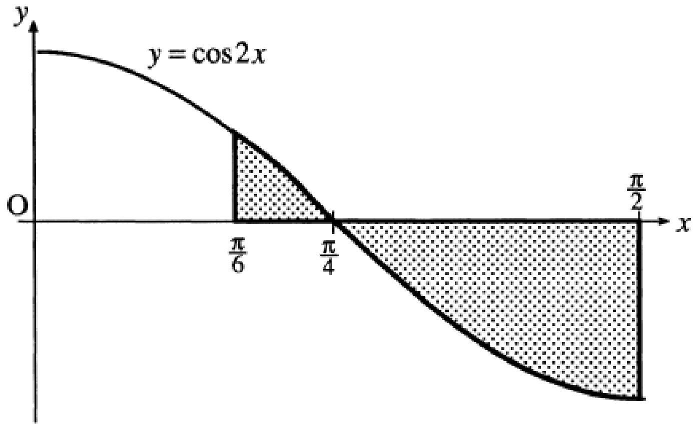
	(*c*) Calculate the shaded area shown in diagram 2.  *Diagram 2*	**7**
2015 SP P2	**9.** A sea-life visitor attraction has a new logo in the shape of a shark fin. The outline of the logo can be represented by parts of • the x axis • the curve with equation $y = \cos(2x)$ • the curve with equation $y = \sin\left(\frac{3}{4}x - \frac{3}{2}\pi\right)$ as shown in the diagram.  Calculate the shaded area.	**6**
2015 P1	**12.** The diagram shows part of the graph of $y = a\cos bx$. The shaded area is $\frac{1}{2}$ unit^2.  What is the value of $\int_0^{\frac{3\pi}{4}} (a\cos bx)\,dx$?	**2**

2002 P1	**10.** (*a*) Find the derivative of the function $f(x)=(8-x^3)^{\frac{1}{2}}$, $x<2$. (*b*) Hence write down $\int \frac{x^2}{(8-x^3)^{\frac{1}{2}}}\,dx$.	**2** **1**
2016 P2	**10.** (a) Given that $y=\left(x^2+7\right)^{\frac{1}{2}}$, find $\frac{dy}{dx}$. (b) Hence find $\int \frac{4x}{\sqrt{x^2+7}}\,dx$.	2 1

Pre 2000 - Further Integration

1	Find $\int \sqrt{1+3x}\,dx$ and hence find the exact value of $\int_0^1 \sqrt{1+3x}\,dx$.	**4**
2	Evaluate $\int_{-3}^{0} (2x+3)^2\,dx$.	**4**
3	Find $\int \left(6x^2-x+\cos x\right)\,dx$.	**4**
4	A sketch of part of the graph of $y=\sin 2x$ is shown in the diagram. The points P and Q have coordinates $(p, 0)$ and $(q, -1)$. (*a*) Write down the values of p and q. (*b*) Find the area of the shaded region. 	**1, 4**
5	(*a*) Evaluate $\int_0^{\frac{\pi}{2}} \cos 2x\,dx$. (*b*) Draw a sketch and explain your answer.	**3, 2**

6 An artist has designed a 'bow' shape which he finds can be modelled by the shaded area below. Calculate the area of this shape. **(6)**



7 (*a*) Show that $(\cos x + \sin x)^2 = 1 + \sin 2x$.

(*b*) Hence find $\int (\cos x + \sin x)^2 \, dx$.

1, 3

8 Differentiate $\sin^3 x$ with respect to x.

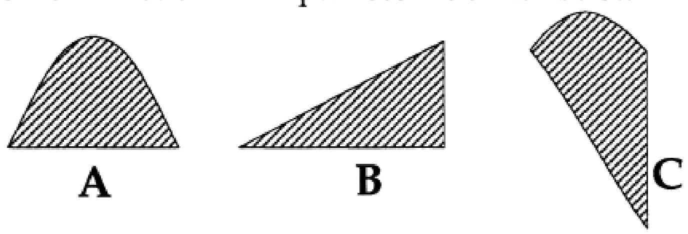
Hence find $\int \sin^2 x \cos x \, dx$.

4

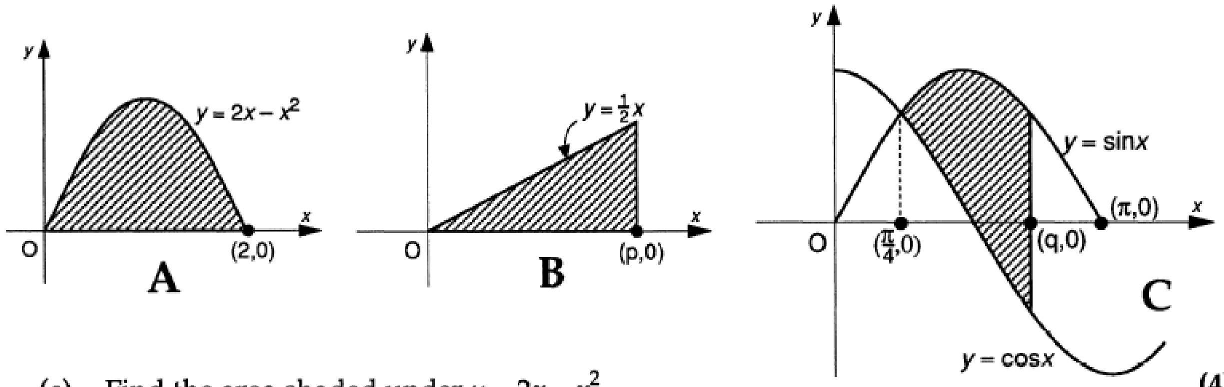
9 (*a*) By writing $\sin 3x$ as $\sin(2x + x)$, show that $\sin 3x = 3\sin x - 4\sin^3 x$. **(4)**

(*b*) Hence find $\int \sin^3 x \; dx$. **(4)**

10 An artist has been asked to design a window made from pieces of coloured glass with different shapes. To preserve a balance of colour each shape must have the **same** area. Three of the shapes used are drawn below.



Relative to *x*,*y*–axes, the shapes are positioned as shown below.



(*a*) Find the area shaded under $y = 2x - x^2$. **(4)**

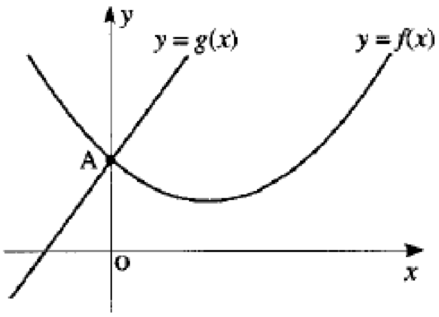
(*b*) Use the area found in part (*a*) to find the value of *p*. **(2)**

(c) Prove that *q* satisfies the equation $\cos q + \sin q = 0.081$ and hence find the value of *q* to 2 significant figures. **(10)**

Differential Equations

2000 P1	**B8.** The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$. If $f'(x) = \sin(3x)$, express y in terms of x. **4**
2001 P2	**10.** A curve for which $\frac{dy}{dx} = 3\sin(2x)$ passes through the point $\left(\frac{5}{12}\pi, \sqrt{3}\right)$. Find y in terms of x. **4**
2002 P2	**8.** A point moves in a straight line such that its acceleration a is given by $a = 2(4-t)^{\frac{1}{2}}$, $0 \le t \le 4$. If it starts at rest, find an expression for the velocity v where $a = \frac{dv}{dt}$. **4**
2014 P2	**9.** Acceleration is defined as the rate of change of velocity. An object is travelling in a straight line. The velocity, v m/s, of this object, t seconds after the start of the motion, is given by $v(t) = 8\cos(2t - \frac{\pi}{2})$. (*a*) Find a formula for $a(t)$, the acceleration of this object, t seconds after the start of the motion. 3 (*b*) Determine whether the velocity of the object is increasing or decreasing when $t = 10$. 2 (*c*) Velocity is defined as the rate of change of displacement. Determine a formula for $s(t)$, the displacement of the object, given that $s(t) = 4$ when $t = 0$. 3
2015 EP P1	**10.** The gradient of a tangent to a curve is given by $\frac{dy}{dx} = 3\cos 2x$. The curve passes through the point $\left(\frac{7\pi}{6}, \sqrt{3}\right)$. Find y in terms of x. **4**
2015 EP P2	**10.** Acceleration is defined as the rate of change of velocity. An object is travelling in a straight line. The velocity, v m/s, of this object, t seconds after the start of the motion, is given by $v(t) = 8\cos(2t - \frac{\pi}{2})$. (a) Find a formula for $a(t)$, the acceleration of this object, t seconds after the start of the motion. **3** (b) Determine whether the velocity of the object is increasing or decreasing when $t = 10$. **2** (c) Velocity is defined as the rate of change of displacement. Determine a formula for $s(t)$, the displacement of the object, given that $s(t) = 4$ when $t = 0$. **3**

Pre 2000 - Differential Equations

1	The graphs of $y = f(x)$ and $y = g(x)$ intersect at the point A on the y-axis, as shown in the diagram. If $g(x) = 3x + 4$ and $f'(x) = 2x - 3$, find $f(x)$.  **4**
2	The curve $y = f(x)$ passes through the point $\left(\frac{\pi}{12}, 1\right)$ and $f'(x) = \cos 2x$. Find $f(x)$. **3**